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MODELLING OF FATIGUE-TYPE SEISMIC DAMAGE FOR NUCLEAR POWER PLANTS

TAMÁS JÁNOS KATONA

University of Pécs
Nuclear Power Plant Paks Ltd.
Hungary
e-mail: katonat@npp.hu

Abstract

Assessment of seismic safety of nuclear power plants requires convolution of plant fragilities with seismic hazard curves. In this paper, an option for representing the uncertainty of seismic fragility is outlined based on the interval and *p*-box concept. The conditional probability of failure is developed as a function of the cumulative absolute velocity. The dependence of the cumulative absolute velocity on the strong motion parameters is analysed. It is demonstrated that the cumulative absolute velocity is an appropriate damage indicator for fatigue failure mode. Relation between cumulative absolute velocity of failure and various failure theories is established in this paper.

1. Introduction

Seismic probabilistic safety assessment of plenty of nuclear power plant (NPP) shows that the earthquakes may be the dominating contributors to the core damage, i.e., to the overall risk. These results indicate the vulnerability of the nuclear power plants against earthquakes. On the other hand, experiences show that plants survive much larger earthquakes than those considered in the design base, as it was the case of Kashiwazaki-Kariwa NPP, where the safety classified

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structures, systems, and components survived the Niigata-Chuetsu-Oki earthquake in 2007 without damage and loss of function [10]. In spite of the nuclear catastrophe of the Fukushima Daiichi plant caused by the tsunami after Great Tohoku earthquake 11th of March 2011, the behaviour of thirteen nuclear unit in the impacted area on the east-shore of the Honshu Island demonstrated high earthquake resistance.

One of the most complex cases for assessing the nuclear power plant safety is the evaluation of the response of the plant to an earthquake load and the risk related with this. The probability safety analysis (PSA) has to demonstrate whether in case of an earthquake the reactor can be shut down, cooled-down, the residual heat can be removed from the core, and the radioactive releases can be limited below the acceptable level. The core damage frequency is the output of the analysis. Well-defined set of plant systems and structures and components (SSCs) are required to be functional during and after the earthquake for ensuring the mentioned above basic safety functions. The frequencies of core damage caused by an earthquake are calculated by plant logic convoluting with component fragilities, see [1] and [8]. Event trees are constructed to simulate the plant system response. Fault trees are needed for the development of the probability of failure of particular components taking into account all failure modes. The hazard is expressed as *complementary probability* = 1 – *cumulative probability function,* i.e., probability that the peak ground acceleration (PGA) exceeds a given value. The fragility is defined as the conditional probability of core damage as a function of horizontal ground motion acceleration **a** – PGA at free surface. Based on the experience, one can conclude that the design basis capacity expressed in terms of peak ground acceleration, which is used for fragility estimation does not provide information about failure in case of a particular earthquake. In [6], considerations were given on the possibility for derivation of conditional probability of failure for cumulative absolute velocity instead of peak ground acceleration.

Beside of the randomness of the resistance of the structure, damage of the structure may depend on the PGA, length of strong motion, frequency content of the vibratory motion, etc.. Therefore, it is rather difficult to validate the fragility as conditional probability of failure versus

PGA. The studies performed by EPRI regarding failure indicators show that the cumulative absolute velocity (CAV) could be better correlated to damage rather than the PGA [3]. The EPRI studies validate the lower bound of standardized CAV for damage of non-engineered structures. U.S. NRC Regulatory Guide 1.166 defines the criteria for exceedance of operational base earthquake level.

In this paper, the basic idea of the calculation core damage frequency due to earthquake is given, and an alternative representation of the uncertainty of the fragility is presented on the basis of [6].

A physical interpretation of the cumulative absolute velocity and its dependence on strong motion parameters and load characteristics relevant for damage indication are discussed in [7]. It is also shown in [7], why the cumulative absolute velocity is an appropriate damage indicator, especially for the fatigue-type damage.

In this paper, the cumulative absolute velocity – which is some kind of measure of the energy of the ground motion obtained from the free-field record – is expressed as a function of basic characteristics of the ground motion, i.e., strong motion duration and power spectra. The cumulative absolute velocity will be linked to the stress or strain amplitudes and cycles affecting the component and causing fatigue type damage. With respect to the earthquake damage, different types of fatigue mechanisms can be considered, e.g., the ultra low-cycle fatigue, low-cycle fatigue. Relation between cumulative absolute velocity of failure and various failure theories, e.g., fatigue failure condition the Bendat narrow-band theory and Dirlik formulation of the random amplitude fatigue failure, and the crack-growth condition has been established in this paper.

2. Basics of the Probabilistic Seismic Safety Analysis

In the probabilistic safety assessment for seismic events (seismic PSA), modelling of complex component behaviour requires Boolean description of sequences leading to failure.

Plant level fragility is obtained by combining component fragilities according to the Boolean-expression of the sequence leading to core damage.

The plant level fragility is defined as the conditional probability of core damage as a function of free field PGA at the site.

Plant level fragilities are convolved with the seismic hazard curves to obtain a set of doublets for the plant damage state.

For evaluation of core damage frequency, the doublets $\{\langle p_{ij},\, f_{ij} \rangle\}$ has to be obtained, where f_{ij} is the seismically induced plant damage state frequency

$$f_{ij} = -\int_{0}^{\infty} f(a')_i \frac{dH_j}{da} da', \tag{1}$$

where p_{ij} is the discrete probability of this frequency $p_{i,j} = q_i p_j$, q_i is the probability associated with of i-th fragility curve, $f(a)_i$, and the p_j is the probability associated with j-th hazard curve, H_j. The fragility curve $f(a)_i$ is the i-th representation of the conditional probability of core damage (plant failure resulting into core damage). The dH_j / da is the probability density function of the applied seismic load expressed in terms of peak ground acceleration, taken from the j-th hazard curve.

According to the Equation (1), the uncertainty in plant level fragility is displayed by developing a family of fragility curves. The weight (probability) assigned to each curve is derived from the fragility curves of components appearing in the specific plant damage state accident sequence, i.e., the process of development of plant fragility starts with identification of failure modes and corresponding conditional probability distribution function for failure for SSCs required for safety.

The procedure is as follows:

– Construct fault tree.

– Convert fault tree logic to Boolean equation.

– Derive minimal cut set from the Boolean equations by using Boolean algebra.

– Calculate top event probability by using the derived minimal cut set and fragility data.

Considering the practical applications of seismic PSA, there are plenty of failure modes to be accounted in the model. Active components typical failure modes are the stretching or loosing, distortion/deformation, drop out of parts, impact/contact, flooding/spraying. Typical failure modes of passive components are breaking, distortion/deformation, drop out of parts, impact/contact, flooding/spraying.

Typical numbers of failure modes for different components are, e.g.,

– Heat exchangers: damage of main body, flange part, heat exchanging tubes, supports, nozzles.

– Valves: malfunction of drive, yoke damage, leakage from valve seat, loss of structural integrity.

– Horizontal pumps: damage of fixes, supports, damage of shaft, shaft joints, mechanical seal, bearing, loss of power, damage of nozzles.

3. Alternative Representation of the Uncertainty of the Fragility

The capacity for a given failure mode is characterized by a log-normal probability distribution with median capacities and logarithmic standard deviations to account for uncertainty in the parameters. Considering the epistemic and aleatory uncertainty, the capacity for a given failure mode can be expressed via median capacity multiplied by two log-normally distributed random variable ε_R and ε_U, representing the uncertainty due to randomness and the epistemic uncertainty, respectively, and with logarithmic standard deviation β_R and β_U, respectively.

According to this, the frequency of failure f' at any non-exceedance probability level Q can be written as follows:

$$f' = \varphi\left[\frac{\ln(a / C_m) + \beta_U \varphi^{-1}(Q)}{\beta_R}\right], \tag{2}$$

where $Q = P(f < f'|a)$, and φ is standard normal distribution function.

Not practical to quantify the seismic PSA models using continuous families of seismic hazard curves and associated equipment fragility distributions. Instead of using families of seismic hazard curves, $\{\langle p_j, H_j\rangle\}$ as well as the set of equipment fragility distribution, $\{\langle q_i, f_i\rangle\}$ point-estimates of hazard and fragility are used with subsequent uncertainty analysis.

The recent practice, the analysis of uncertainties is based on the probability theory: Point estimates are used in combination with Monte-Carlo sensitivity analysis.

Another method for describing and quantifying uncertainty in the model represented by Equation (1) can be based on interval probability or p-box theory. Instead of point estimates, the upper and lower bounds of the distribution functions might be used for replacing the sets $\{\langle p_j, H_j\rangle\}$ and $\{\langle q_i, f_i\rangle\}$ by probability boxes specified by a left side and a right side distribution functions.

Based on [6], for the fragility, the following representation can be applied:

$$\{\langle q_i, f_i\rangle\} \to \left[\overline{F}(x), \underline{F}(x)\right], \tag{3}$$

where $\left[\overline{F}(x), \underline{F}(x)\right]$ are the probability-boxes specified by a left side $\overline{F}(x)$ and a right side $\underline{F}(x)$ distribution functions, and $\underline{F}(x) \le F(x) \le \overline{F}(x)$.

From a lower probability measure $\underline{P}$ for a random variable X, one can compute upper and lower bounds on distribution functions by using formula

$$\overline{F}_x(x) = 1 - \underline{P}(X > x),$$

$$\underline{F}_x(x) = \underline{P}(X \le x). \tag{4}$$

It is often convenient to express a p-box in terms of its inverse functions d and u defined on the interval of probability levels $[0, 1]$. The function u is the inverse function of the upper bound on the distribution function and d is the inverse function of the lower bound. These monotonic functions are bounds on the inverse of the unknown distribution function $\underline{F}$

$$d(p) \ge F^{-1}(p) \ge u(p), \tag{5}$$

where p is probability level.

In this case, the only information needed (or available) is that

$$\underline{P}_{fail} = \begin{cases} 0, & \text{if } x \le \overline{x}, \\ 1, & \text{otherwise}, \end{cases}$$

$$\overline{P}_{fail} = \begin{cases} 0, & \text{if } x \le \overline{x}, \\ 1, & \text{otherwise}, \end{cases} \tag{6}$$

where p-box might be defined in case when the minimum, maximum or median, and/or other percentiles of failure distribution are known.

The most trivial case for the use of p-box can be while performing the screening according to ruggedness of the component. The rugged components might be described by p-box with a lower bound $\underline{x}$ (PGA or any other damage indicator) below of that no failure may occur and an upper bound of $\overline{x}$ above that the failure will occur for sure.

The probability bounds might be calculated for cases in which, the distribution family is specified by interval estimates of the distribution parameters.

Probability bounds can be calculated for distribution families using only interval estimates for the parameters or having information only on {min, max} or {min, max, mode} or {min, max, mean} of the variable. Upper and lower bounds on parametric values can be provided, typically from expert elicitation

If the bounds on mean, μ and standard deviation σ are known, bounds on the distribution can be obtained by computing the envelope of all log-normal distributions L that have parameters within the specified intervals

$$d(p) = \max_{\alpha} L_{\alpha}^{-1}(p),$$

$$u(p) = \min_{\alpha} L_{\alpha}^{-1}(p), \tag{7}$$

where $\alpha \in \{(\mu, \sigma) | \mu \in [\mu_1, \mu_2], \sigma \in [\sigma_1, \sigma_2]\}$, see Figure 1.

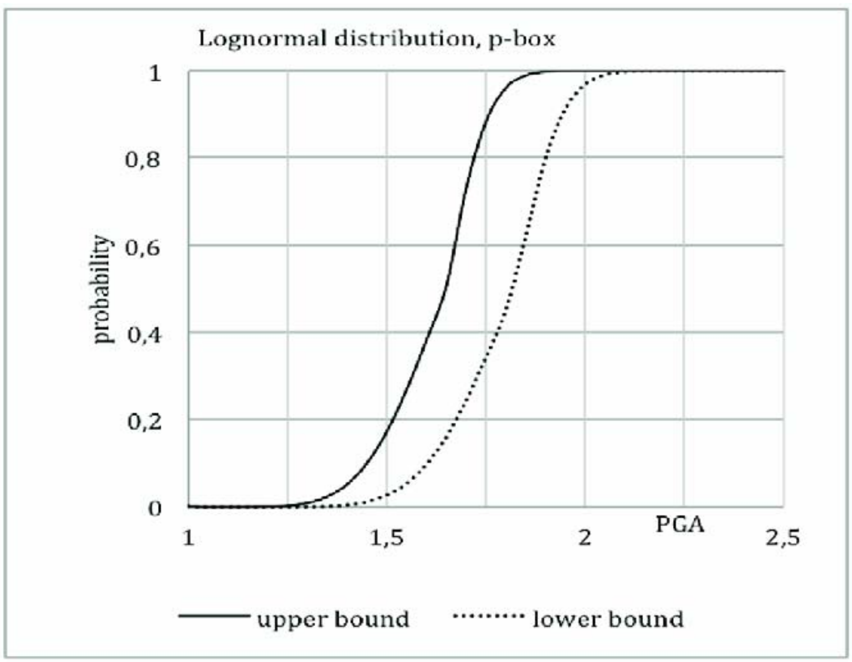

Figure 1. The *p*-box for log-normal distribution.

Real benefit from this type of representation of probability distribution might be obtained, if the fragility of a particular failure mode of a component is known approximately only, small sample size of damage histories, inconsistency of data, or the modelling of failure component is uncertain (e.g., if the set of possible failure modes might be incomplete).

Explicit numerical methods exist for computing bounds on the result of addition, subtraction, multiplication, and division of random variables, when only bounds on the input distributions are given (see, e.g., [5] and [12]). These algorithms have been implemented in software and have been extended to transformations such as logarithms, and square roots, other convolutions such as minimum, maximum, and powers, and other dependence assumptions.

3. Fragility Versus Cumulative Absolute Velocity

3.1. The cumulative absolute velocity as fragility parameter

CAV is calculated as simple integral over the time history of absolute value of acceleration component

$$CAV = \int_0^T |a(t)|\, dt. \tag{8}$$

The standardized CAV is calculated applying a noise-filter for the amplitudes less than ±0.025g [3]. CAV is depending on several parameters of the strong motion: duration, PGA, frequency content.

As it is indicated in [6], the probability of damage/failure is depending on a load vector $\overline{X} = (x_1, x_2, \ldots)$ rather than on a single parameter.

$$P_{fail} = \int_R h(x_1, x_2, \ldots) P(x_1, x_2, \ldots) dx_1 dx_3 \ldots, \tag{9}$$

where $h(x_1, x_2, \ldots)$ represents the hazard, i.e., it is the probability density function of applied loads in terms of CAV and $P(x_1, x_2, \ldots)$ denotes the conditional distribution function of failure.

This approach might seem theoretically precise, however definition of the dependence of fragility on the components of the load vector requires enormous effort. The characterization hazard should also correspond to the description of fragility.

It seems to be interesting to establish a method for fragility modelling based on use of CAV as a non-negative single load parameter $x \geq 0$. The considerations below are made on the basis of [6].

For the sake of simplicity of writing, CAV will be denoted below simple by x. Equation (9) can be rewritten as follows:

$$P_{fail} = \int_0^\infty h(x)P(x)dx. \tag{10}$$

Assuming that, if a failure occurs for a value of CAV equal to x, then it is occurs for all values larger than x.

In this case, the conditional probability distribution function $P(x)$ coincides with the cumulative probability distribution function of the failure load parameter λ, i.e., of the smallest value of the load parameter that the structure is unable to withstand [6],

$$P(x) = P(\lambda \leq x). \tag{11}$$

From the equation above, the average value of the failure load parameter can be calculated. The average CAV-value of failure

$$\overline{\lambda} = \int_0^\infty x' \frac{dP(x')}{dx} dx'. \tag{12}$$

With other words, for the effective use of CAV in fragility analysis, the value $\overline{\lambda}$ has to be evaluated from the empirical data (damages of earthquakes, fragility tests) for all type of SSCs and failure modes.

3.2. Interpretation of the physical meaning of the CAV

As it has been shown in [6], the value of CAV is varying within wide range depending on several parameters of the ground motion: PGA, duration, T, and frequency content of the random motion. However, these dependences except of the dependence on T are not obvious and not explicit. It is reasonable to define the dependence of the CAV on the strong motion parameters.

Let consider the Equation (8) and apply the mean value theorem for the integral. The $|a(t)|$ is an integrable function and its mean value on T is equal to $E\{|a(t)|\}$. The Equation (8) can be rewritten as follows:

$$CAV = \int_0^T |a(t)|dt \cong T * E\{|a(t)|\}. \tag{13}$$

Thus, the CAV can be considered as product of two random variables, the duration of strong motion T and the mean of absolute value of ground acceleration time history. Generally, the variables T and $E\{|a(t)|\}$ might not be independent.

The strong motion acceleration time history can be written in form $a(t) = I(t)x(t)$, where $I(t)$ is a window-function on $[0, T]$ interval, i.e., $I(t) \equiv 0$ if $t = 0$ and $t = T$ and outside of interval and $I(t) > 0$ within the interval. It is assumed that $x(t)$ is a stationary normal random process. However, $a(t)$ is a non-stationary normal process. For the sake of simplicity, let us assume that $a(t)$ is a stationary normal random process with zero mean and probability density function $f_a(a)$ and autocorrelation function $R(\tau)$. In this case, the random process $z(t) = |a(t)|$ has the density function, $f_z(z) = 2f_a(z)U(z)$, and its mean value is as follows (see [9]):

$$E\{|a(t)|\} = \int_{-\infty}^{\infty} |a| f_a(a)da = \sqrt{\frac{2}{\pi}R(0)}, \tag{14}$$

where $R(0)$ is the autocorrelation function of $a(t)$ at $\tau = 0$.

We can write further for $R(0)$ that

$$R(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{aa}(\omega)d\omega, \tag{15}$$

where $S_{aa}(\omega)$ is the power spectral density (PSD) function of $a(t)$.

The power spectral density function $S_{aa}(\omega)$ of the earthquake ground motion acceleration is showing band-limited or even narrow-band character. Since we intend to explain the qualitative features of the CAV, we may assume that $a(t)$ is an ideal band-limited process with PSD

$$S_{aa}(\omega) = \begin{cases} S_0, & \text{if } \omega_1 \le \omega \le \omega_2, \\ 0, & \text{elsewhere.} \end{cases} \tag{16}$$

This assumption is based on NUREG-0800, where the one-sided PSD of the horizontal ground motion acceleration time history is linked the Regulatory Guide 1.60 standard response spectrum [7].

It is obvious that, the excitation energy is concentrated within a narrow frequency range. Thus, the $R(0)$ can be written as follows:

$$R(0) = \frac{1}{2\pi} S_0 \Delta\omega, \tag{17}$$

where $\Delta\omega = \omega_2 - \omega_1$ is the bandwidth. The Equations (8) and (13) can be rewritten as follows:

$$CAV = T\sqrt{\frac{1}{\pi^2} S_0 \Delta\omega}. \tag{18}$$

Introducing the median frequency ω_c;

$$\omega_c = \omega_1 + \frac{\omega_2 - \omega_1}{2}. \tag{19}$$

In addition, the number of load cycles during the strong motion, N

$$T\omega_c = 2\pi N, \tag{20}$$

the CAV can also be expressed as follows:

$$CAV = \frac{2}{\omega_c} N \sqrt{S_0 \Delta\omega}. \tag{21}$$

If the $a(t)$ is band-limited, it can be represented via sum of sine functions

$$a(t) = \sum_{i=1}^{n} A_i \sin(\omega_i t + \phi_i). \tag{22}$$

The energy of $a(t)$ should be distributed according to the following equation:

$$E\{a^2(t)\} = \sum_{i=1}^{n} \frac{A_i^2}{2}. \tag{23}$$

Furthermore, the frequencies ω_i should be within $\delta\omega$ intervals and

$$S_{aa}(\omega_i)\delta\omega = \frac{A_i^2}{2}. \tag{24}$$

For the sake of simplicity, let us represent the excitation by a single sine with median frequency ω_c and $2S_0\Delta\omega = A_c^2$, we obtain

$$CAV = \frac{1}{\pi} T A_c = 2\frac{1}{\omega_c} N A_c. \tag{25}$$

On the basis of above considerations, the following conclusion can be drawn:

– The CAV is proportional to the product of strong motion duration and average energy (RMS) of the strong motion acceleration time history $a(t)$, as it shown by Equation (13). This result is rather trivial.

– The CAV should be an adequate damage indicator, since it is reflecting the main parameters of damage phenomena. The CAV depends on the strong motion duration, T, number of load cycles, N, and median frequency, ω_c, and amplitude of the alternating load, which is proportional to the ground motion acceleration amplitude, i.e., A_c.

– The higher the mean frequency of the excitation is, the less will be the possibility of a damage, which corresponds to the observations.

3.3. CAV as damage indicator for fatigue failure mode

Several damage mechanisms due to cyclic earthquake loads might be in place:

– Ultra-low cycle fatigue.

– Low-cycle fatigue without crack or with pre-existing crack.

A straightforward calculation of the fatigue damage to a structure consists of four basic steps:

(1) Calculate the (nonlinear) time-history response of the structure to an earthquake loading.

(2) Extract the response quantities of interest.

(3) Convert the time history response to an equivalent series of loading cycles.

(4) Calculate the fatigue damage of the equivalent cyclic responses.

Fatigue damage estimation involves the cycle counting of equivalent stress ranges and accumulation of fatigue damage from each cycle.

With mechanical equipment, the cycle amplitudes are generally constant and known and the fatigue limit is directly determined from experiments. However, seismic loads are not made up of complete, consistent cycles. Typical seismic response time histories exhibit varying amplitudes, mostly partial cycles, and no complete symmetric cycles.

After estimating an equivalent stress range distribution either in the time domain or in the frequency domain, the linear Palmgren–Miner rule is used to predict the damage per cycle as

$$D_i = \frac{1}{N_i}, \tag{26}$$

where D_i is the damage for cycles of magnitude *i,* and N_{fi} is the number of cycles to failure at level *i*. The total damage to a member over the complete cycling history is then estimated as

$$FDI = \sum_{i=1}^{n} \frac{N_i}{N_{fi}}, \tag{27}$$

where FDI is the fatigue damage index, or total damage to the element due to the cyclic load, n is the number of different cycle amplitudes in the loading history, and N_i is the number of cycles at amplitude i.

Values of FDI greater than or equal to 1.0 indicate a low-cycle fatigue fracture of the structure.

In order to use the measured seismic response to calculate fatigue damage, it is necessary to convert the time history to a series of varying amplitude cycles. The rain-flow method is most commonly used for converting random stressors to cycles.

Let us consider a free-field mounted one-degree-of-freedom (ODF) structure, with m, mass, k, stiffness, c, damping, ω_0, resonance frequency, $\xi = {}^{c}\!/_{2\sqrt{km}}$, and transfer function $H(\omega, \xi, \omega_0)$. Assuming linear stress strain relationship, the stress level $s(t)$ is directly proportional to the relative displacement level $z(t)$, i.e., $s(t) = kz(t)$. Calculating the relative displacement response of the ODF system to the sinusoidal excitation with $A_c \sin(\omega_c t)$, the stress level might be defined.

Standard fatigue model might be applied for the definition of the number of cycles to failure for the calculated stress amplitude.

For the ODF system, the CAV to fail can be calculated by using Equations (13), (14), and (15). In the case of generic base excitation $a(t)$, the response of the ODF in terms of stress level will be

$$S_{ss}(\omega) = k^2\omega^2 |H(\omega, \xi, \omega_0)|^2 S_{aa}(\omega). \tag{28}$$

Equation (28) can be generalised introducing the transfer function $H_{ay}(\omega)$ between ground motion acceleration $a(t)$ and any response quantity of interest y (e.g., stress, strain, displacement)

$$S_{yy}(\omega) = |H_{ay}(\omega)|^2 S_{aa}(\omega), \tag{29}$$

For the definition of the fatigue failure based on the spectral representation of the stress history $S_{ss}(\omega)$, the moments m_i of the power spectral density have to be calculated for the definition of the stress range, number of zero-crossing, number of amplitude maxima

$$m_i = \int_0^{\infty} \omega^n G_{ss}(f\omega) d\omega, \tag{30}$$

where $G_{ss}(\omega)$ is the one-sided PSD corresponding to $S_{ss}(\omega)$. Thus, the following equation can be written:

– for the root-mean-square value $RMS = m_0$,

– for the zero-crossing

$$E[0] = \sqrt{m_2 / m_0}; \tag{31}$$

– for the peak-rate

$$E[P] = \sqrt{m_4 / m_2}. \tag{32}$$

Utilising Equation (29) and assuming narrow-band feature for ground motion acceleration as it is expressed by the Equation (16), Equation (30) can be rewritten as follows:

$$m_i = \int_0^{\infty} \omega^n \left|H_{ay}(\omega)\right|^2 S_{aa}(\omega) d\omega, \tag{33}$$

$$m_i = S_0 \int_{\omega_1}^{\omega_2} \omega^n |H_{as}(\omega)|^2 d\omega. \tag{34}$$

For the RMS, the following equation can be derived:

$$m_0 = S_0 \int_{\omega_1}^{\omega_2} |H_{as}(\omega)|^2 d\omega. \tag{35}$$

It means that the excepts of the $RMS = m_0$, all interested values, $E[0]$ and $E[P]$ are independent form the excitation.

For the calculation of fatigue failure condition, the Bendat narrow-band theory can be used [2]. It is assumed, that the probability density function of the peaks for the narrow band signal can be approximated by Rayleigh distribution. Thus, the equation can be written as follows:

$$FDI = \sum_{i=1}^{n} \frac{N_i}{N(S_i)} = \frac{S_t}{\kappa} \int_0^{\infty} S^b p(S) dS,$$
$$FDI = \frac{E(P)T}{\kappa} \int_0^{\infty} S^b \left[\frac{S}{4m_0} \exp\left(\frac{-S^2}{8m_0} \right) \right] dS, \tag{36}$$

where $N(S_i)$ is the number of cycles of stress range S occurring in T seconds, N_i is the actual counted number of cycle, $S_t = N$, the total number of cycles equal to $\{E[P]T\}$, $E[P]$ is the number of peaks per second. Parameters κ and b are the materials constant defining the fatigue curve. In the Equation (36), the m_0 is depending on the input excitation power spectral density function, which is assumed to be equal to S_0.

In the Equation (36), the m_0 can be expressed via Equations (18) and (21). Thus, the link between CAV and fatigue failure condition is established.

Similarly, the Dirlik solution [4] for the $p(S)$ probability density function can be linked to the CAV of ground motion exciting the structure.

The number of stress cycles of range $N(S)$ can be calculated via

$$N(S) = E[P]Tp(S). \tag{37}$$

Dirlik solution for the probability density function is as follows:

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2\sqrt{m_0}}, \tag{38}$$

where D_1, D_2, D_3, and R are parameters depending on m_0, m_1, m_2, and m_4, but not depending on the input excitation power spectral density, if the excitation is narrowband.

The Z is depending on the RMS of the input excitation, which can be expressed by CAV via Equations (17) and (18).

Thus, the final results of the calculation of the fatigue failure can be correlated to the cumulative absolute velocity of the ground motion excitation.

There are other fatigue failure theories, which can be correlated to the ground motion excitation via CAV.

Assuming that the failure mode is the low-cycle fatigue, the well-known Coffin-Manson relation for low-cycle fatigue can be written

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N)^c, \tag{39}$$

where $\Delta\varepsilon_p/2$ is the plastic strain amplitude, ε'_f is the fatigue ductility coefficient, $2N$ is the number of reversals, or simple the N cycles, and c is an empirical constant ranging from – 0.5 to – 0.7.

For the sake of simplicity, the represent the excitation by a single sine with median frequency ω_c, and $2S_0\Delta\omega = A_c^2$. Assuming that the parameter interested is

$$\frac{\Delta\varepsilon_p}{2} = |H|A_c, \tag{40}$$

where $|H|$ is the amplification of the transfer. For the CAV to fail can be written using Equation (25) as follows:

$$CAV_{fail} = \frac{2\varepsilon'_f}{\omega_c} N^{(1+c)}. \tag{41}$$

Thus, the CAV to fail is connected via Equation (41) to the failure criteria due to the low-cycle fatigue.

Failure of a material due to fatigue may be viewed on a microscopic level. The first phase is the crack initiation. The crack may be caused by surface scratches due to the handling, or tooling of the material, or intersecting the surface as a result of previous cyclic loading. The second phase is the crack propagation: The crack continues to grow during this stage as a result of continuously applied stresses. The final phase is the failure, which occurs when the material that has not been affected by the crack cannot withstand the applied stress. For example, the Newman's stress intensity solution can be used to calculate the RMS stress intensity factor range ΔK_{RMS}, see, e.g., [11].

$$\Delta K_{RMS} = (\sigma_{\max, RMS} - \sigma_{\max, RMS})\sqrt{\frac{\pi a}{Q}} M_e, \tag{42}$$

where a is the crack depth and Q is the elastic shape factor for an elliptical crack, and M_e is the elastic magnification factor. The maximum and minimum stress RMS values can be calculated utilising the moments of the power spectral density function of the stress response, see Equation (30). The latter can be linked to the power spectral density of the excitation via Equation (28).

4. Conclusion

Seismic probabilistic safety assessment became recently high importance. Reliable methods for justification of the plant safety are needed for the cases, when earthquake hits the plants.

One of the basic issues of seismic PSA development is the definition of component and plant fragilities. Sparse statistical information exists on behaviour of complex structures/machines under earthquake loads. Fragility test of components might be very expensive. The experimental data does not provide information on all possible failure modes. Epistemic uncertainty in the failure modelling might be substantial.

One possible way for the seismic PSA improvements might be the utilization of bounding approach as outlined in the paper. A bounding approach to risk analysis extends and complements traditional probabilistic analyses when analysts cannot specify precise parameter values for input distributions or point estimates in the model, precise probability distributions for some or all of the variables in the risk model, nature of dependencies between variables or even the exact structure of the risk model. Upper and lower bounds on parametric values can be provided, typically from expert elicitation. There are several advantages of utilization of interval and *p*-box description of uncertainties. Probability bounds can be calculated for distribution families using only interval estimates for the parameters or having information only on {min, max} or {min, max, mode} or {min, max, mean}. Explicit numerical methods exist for computing bounds on the result of addition, subtraction, multiplication, and division of random variables when only bounds on the input distributions are given. These methods are successfully used in other areas of risk analysis.

In the seismic PSA practice, the component fragility development is based on the design information anchored into PGA. Other representation of load, for example, using cumulative absolute velocity as load parameter may improve the calculation of probability failure. As it is shown in the paper, the CAV-value correlated to the failure can be used as the failure load parameter. It is also shown in the paper, that the CAV is an adequate damage indicator since it is reflecting the main parameters of damaging processes, e.g., the CAV is proportional to load cycles causing low-cycle fatigue type damage. In this paper, the dependence of the CAV on the strong motion duration, T, number of load cycles, N, and median frequency, ω_c, and amplitude of the alternating load, A_c (the ground motion) is demonstrated.

Based on the interpretation of the CAV of the equipment given in the paper, the CAV can be correlated to the failure criteria for fatigue. The CAV can also be linked to the failure criteria of the random amplitude, frequency-domain fatigue analysis, as well as to the low-cycle fatigue failure criteria. A correlation between CAV and stress intensity factor range can also been established.

Having this correlation, one can assess the condition of the equipment if an earthquake happen, which may contribute to the quick assessment of the plant condition after an earthquake.

There is an advantage of the use of the CAV for damage indicator, since the CAV is calculated nearly real-time form the easy to measure free-field acceleration signal. We may define for the critical equipment the correlation between fatigue failure and CAV to fail. Having this correlation, one can assess the condition of the equipment if an earthquake happens. The presentation of the CAV given above allows also the correlation of the CAV to the theories of frequency-domain fatigue analysis taking into account the narrow-band character of seismic excitation.

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